WHITE PAPER

CLARIFYING NONLINEAR ANALYSIS

GT STRUDL

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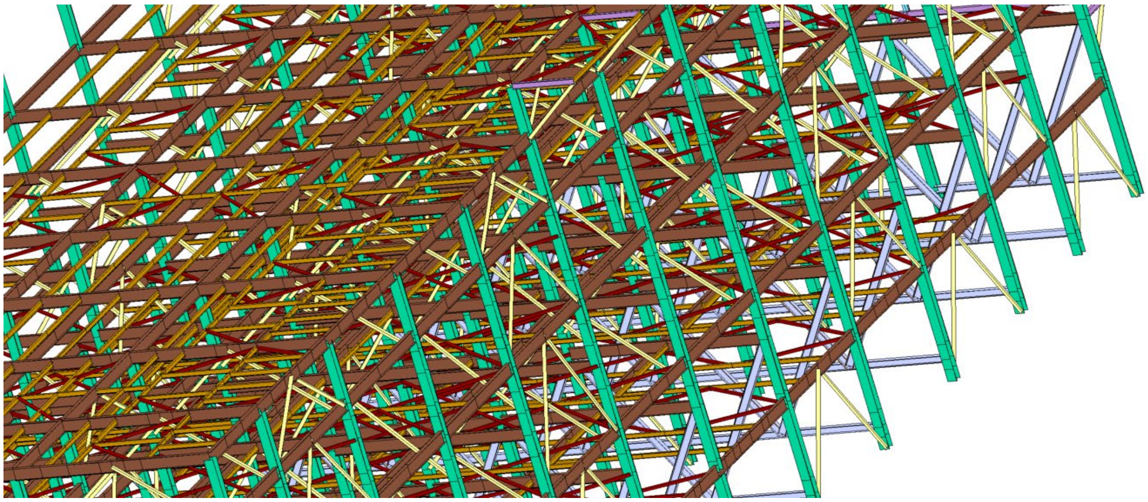
1.1. DEFINITION OF A PROBLEM



“Non-Linear Analysis” is a term frequently thrown out in the world of structural analysis. The reality is that there is no “one button” in any software product that will do non-linear analysis “exactly” the way one may want, because the term is relative. Non-linear could mean structural geometric non-linearity only, material non-linearity only, support condition non-linearity, or a combination of all.

The reason why the term “non-linear” may have become more popular in the world of steel design is because of the analysis requirements introduced in Chapter C of the AISC 14th Ed. Code. This chapter is entitled “Design for Stability,” and Cl. C2 of this code states:

“The analysis shall be a second-order analysis that considers both P-Δ and P-δ effects…”

Second-order analysis is a form of non-linear analysis. This is where most of the confusion comes into the picture.

In a busy production environment where design and drawing deliverables need to be produced, the tendency is to utilize the available tools to get designs out of the door. When any new major code analysis requirements are introduced, engineers really need to step back to see the “why”, “where”, “how,” and “what” before attempting to apply these new requirements. This whitepaper describes how structural engineers can grasp the lateral stability requirements introduced in the AISC 15th Ed. code and use them appropriately and with more confidence without having to do any complicated and elaborate hand verification calculations.

2. INTRODUCTION

The direct analysis procedure was introduced in the AISC 360-05/IBC 2006, Appendix 7 structural steel design code. It is significantly different from the old approach of analyzing steel frames to comply with the older AISC/ASD 9th Ed. Steel Design Code. Analysis and design were thought of as two independent tasks to be performed prior to the introduction of this procedure. This caused considerable discussion in the engineering community.

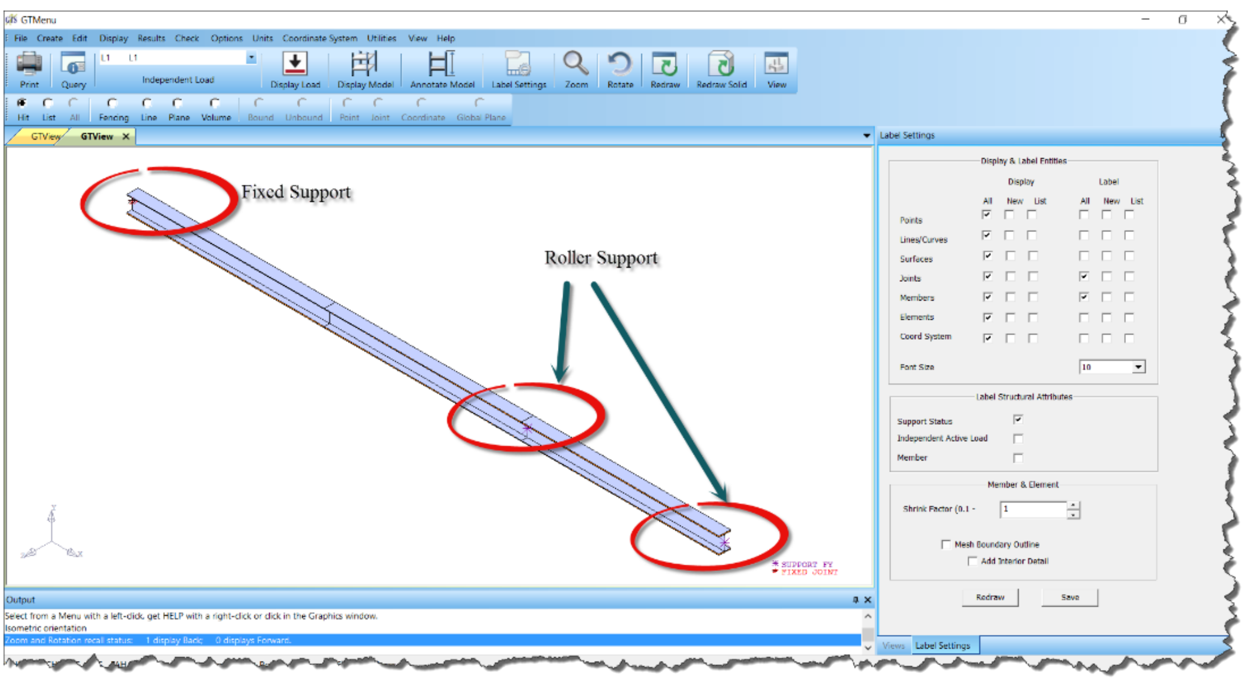
New design codes usually are not used by practitioners until a local building code starts mandating it. This is something that code authors would not like to hear but proved to be a good time buffer for many software companies/developers to study the implications to their software coding and design. AISC 360-05/IBC 2006 allowed engineering professionals to design their structures using the following procedures:

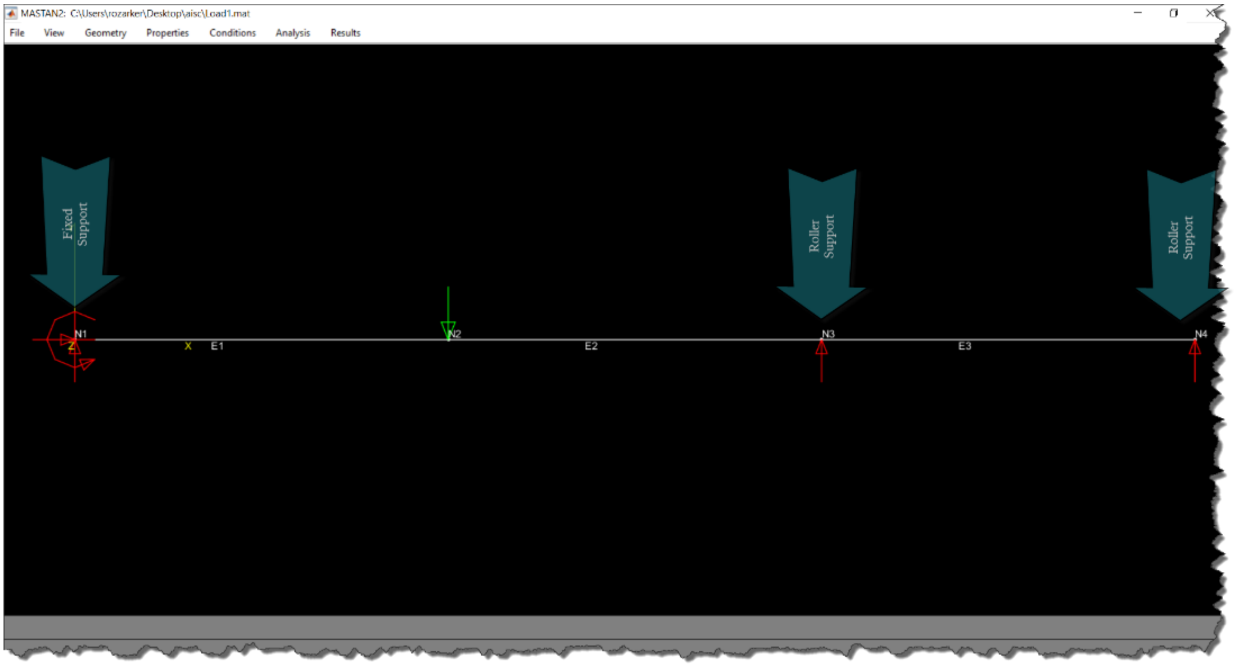
- Effective Length Method – Discussed in Chapter C
- AISC 360-05/IBC 2006, Appendix 7. No matter which method an engineer chooses to consider, he/she needs to be aware of the requirements of chapter C of this design code, particularly; P-Δ and P-δ Stability Check, Geometric Imperfections, and Stiffness Reduction.

3. STEPPING BACK INTO A VIRTUAL STRUCTURES LABORATORY

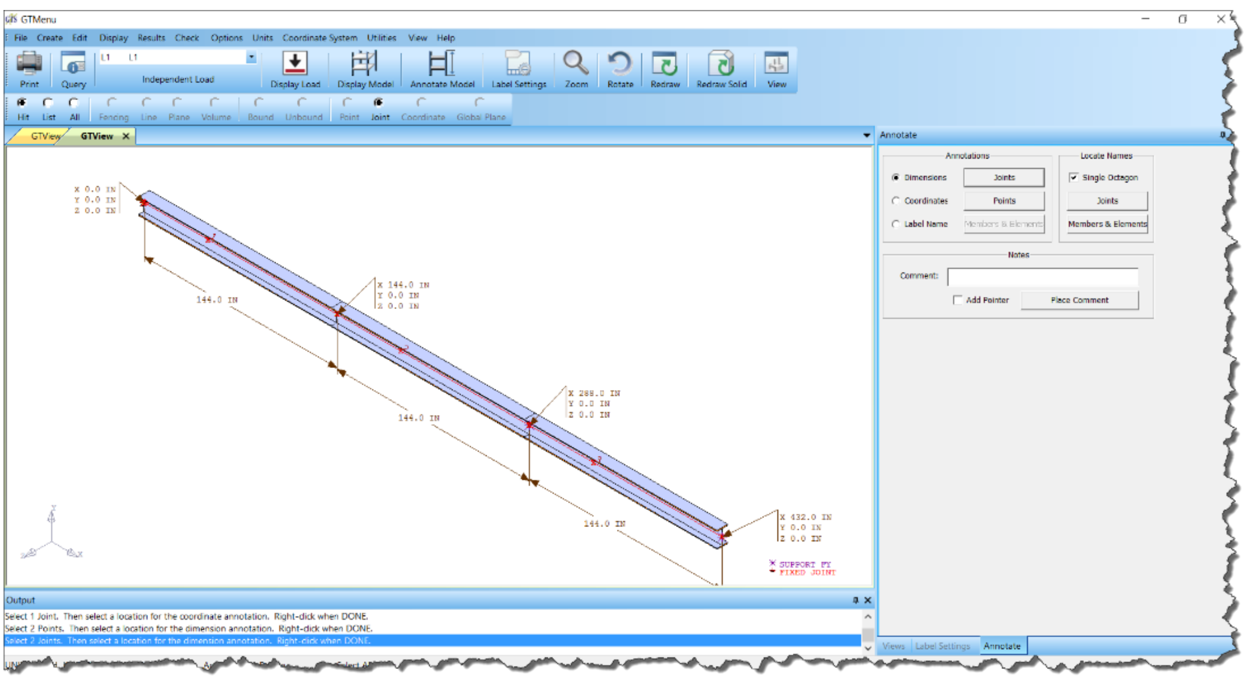
Some engineering concepts can be better grasped by performing experiments. It would be very expensive for any of us to perform these types of experiments in a real structures laboratory in an academic setting. Engineers should use their analysis software to perform these experiments/calculations to not only better understand these code provisions, but also to verify that their software is meeting the requirements and performing the way they expect. We will use GT STRUDL in this case as a virtual structures laboratory on our desktop, and verify the results against Mastan2, v3.5 which was created by Dr. Ronald D. Ziemian (Bucknell University) and the late Dr. William McGuire (Cornell University), the authors/contributors of this code requirement.

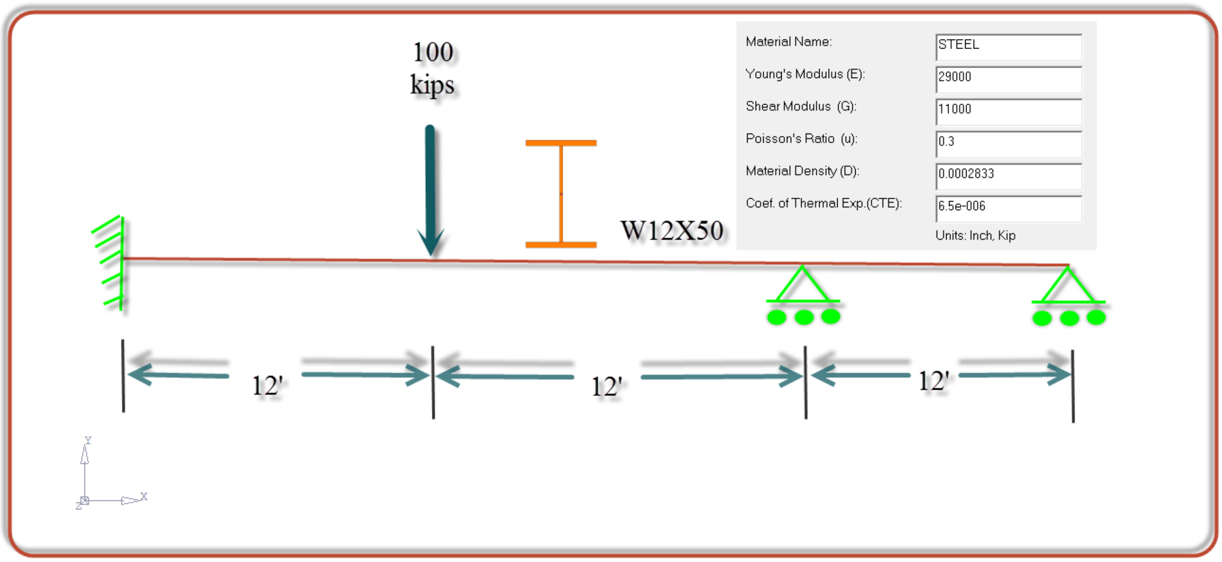
Let us start off with a beam with a fixed support on the left and two roller supports on the right.



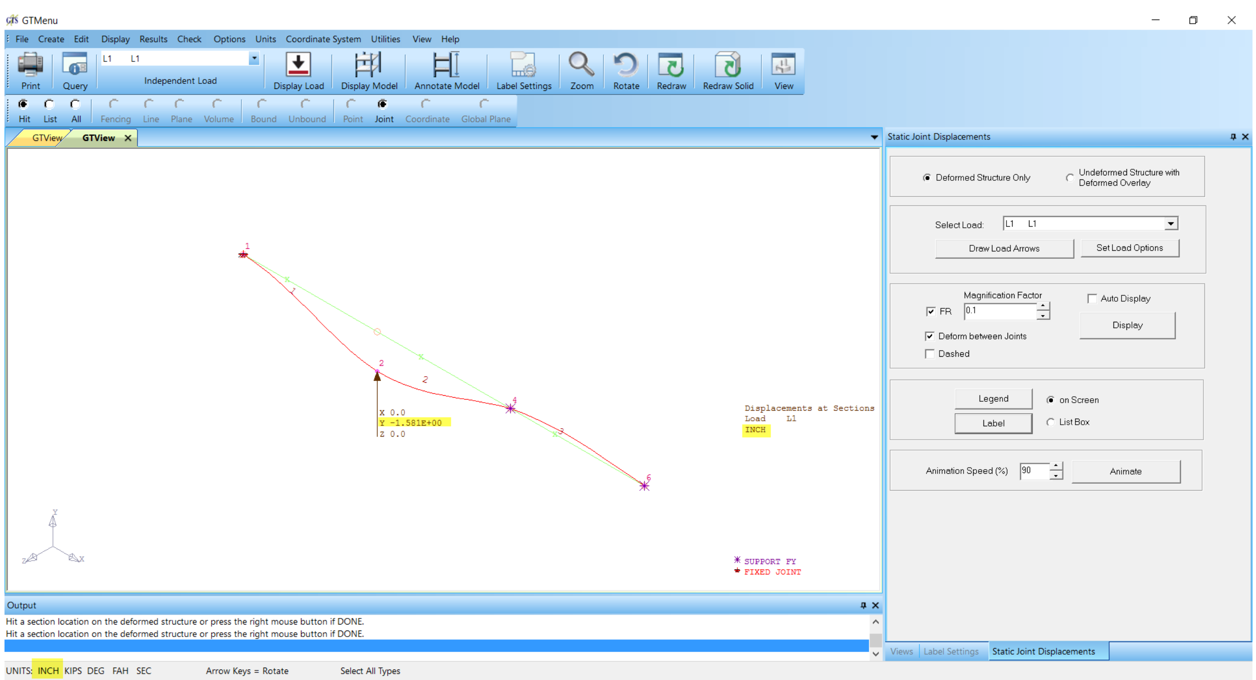


First, let us load the beam with a 100-kip unit load.

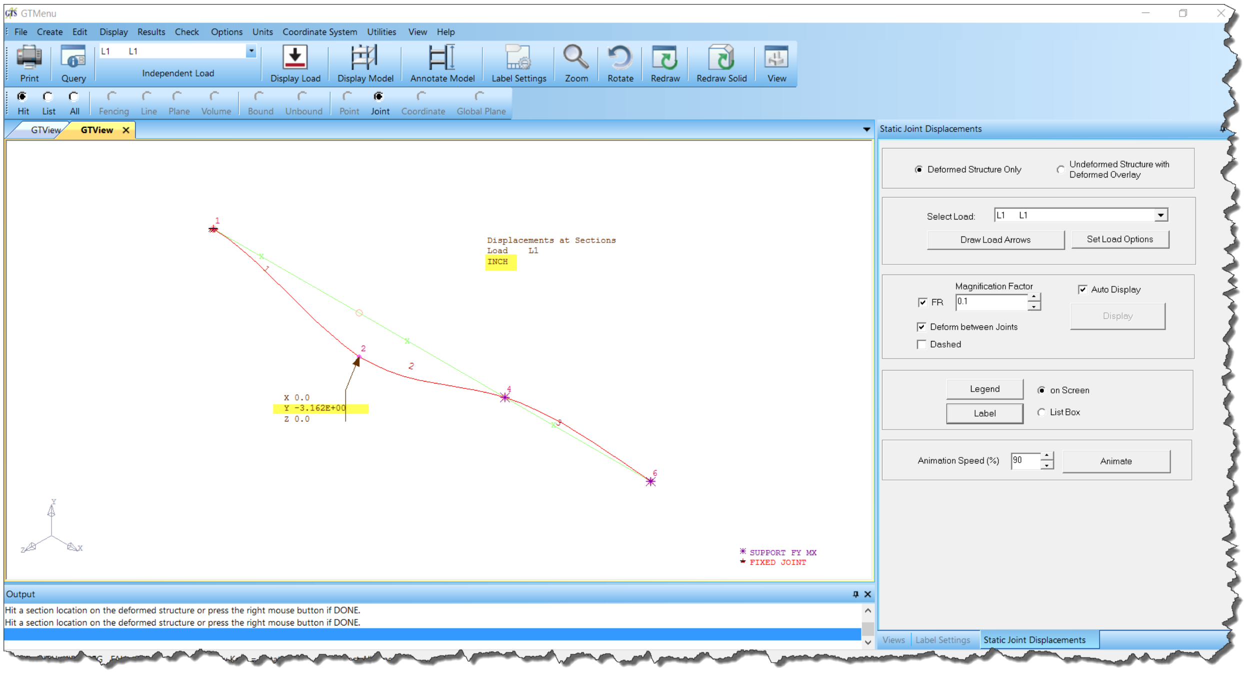




Looking at the results of this unit load, we see a 1.58" deflection at node 2, the point of load application.

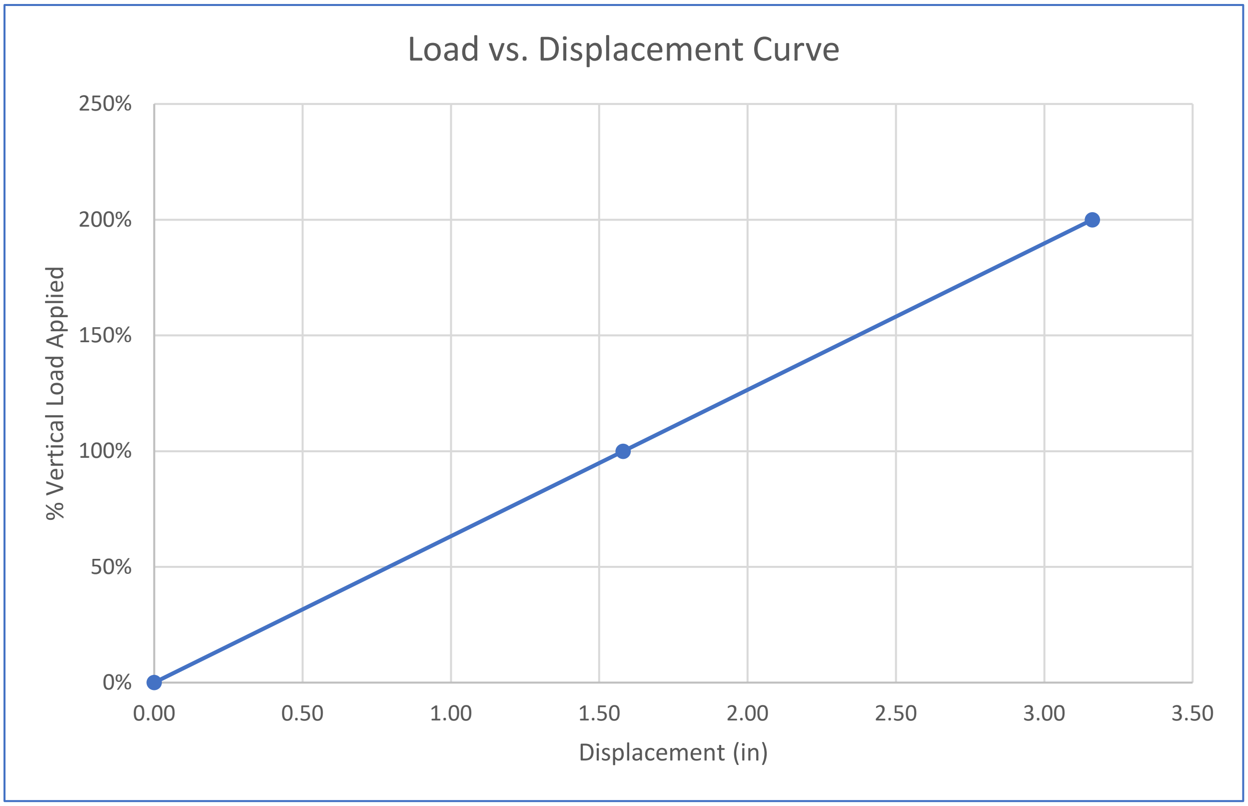


Now, let us double the load and see what results we can obtain.



The deflection at node point 2 is now 3.16 in.

The vertical load vs displacement curve looks plotted using the above data shows a linear behavior.

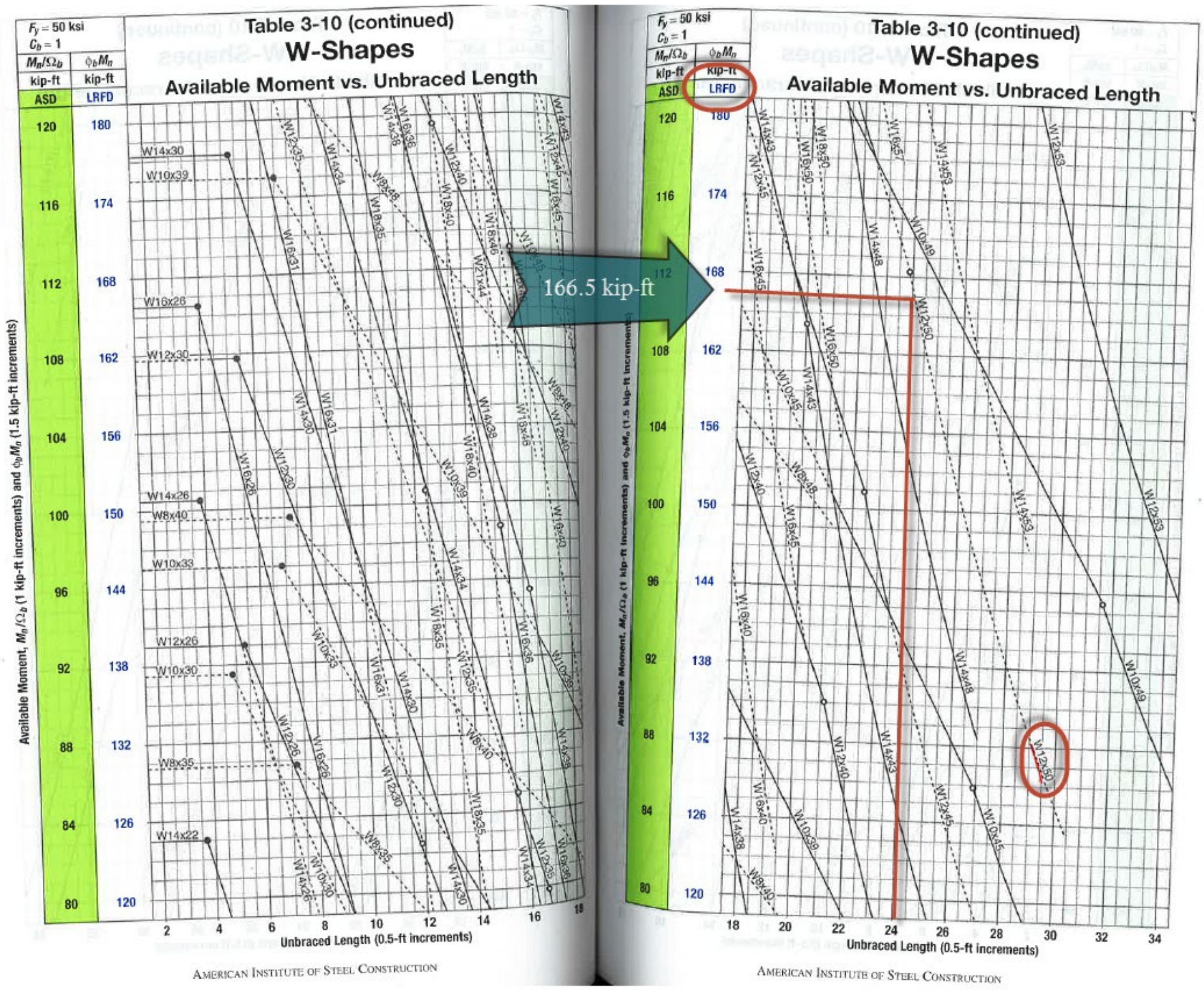


This means that while the beam will eventually fail a code check, it will continue to have linear displacements as the load increases well beyond the code allowable load.

This is not a good virtual structures laboratory as it does not reflect the actual behavior we would see in a real-world lab or failure condition. We really want to see how and when the beam will fail.

To solve this problem, let us first investigate the capacity of the cross-section that we have selected and compare that to the allowable bending moments.

If you look at the AISC 15 Ed. available moment chart for a W12x50, you will notice that this beam can take about 166.5 kip-ft of bending moment, but the moment generated by the 200-kip loading is 718 kip-ft.



This produces a design unity ratio of 718/166.5 or 4.31.

This same design ratio is observed when a code check is performed for member #1.

GT STRUDL Code Check Results

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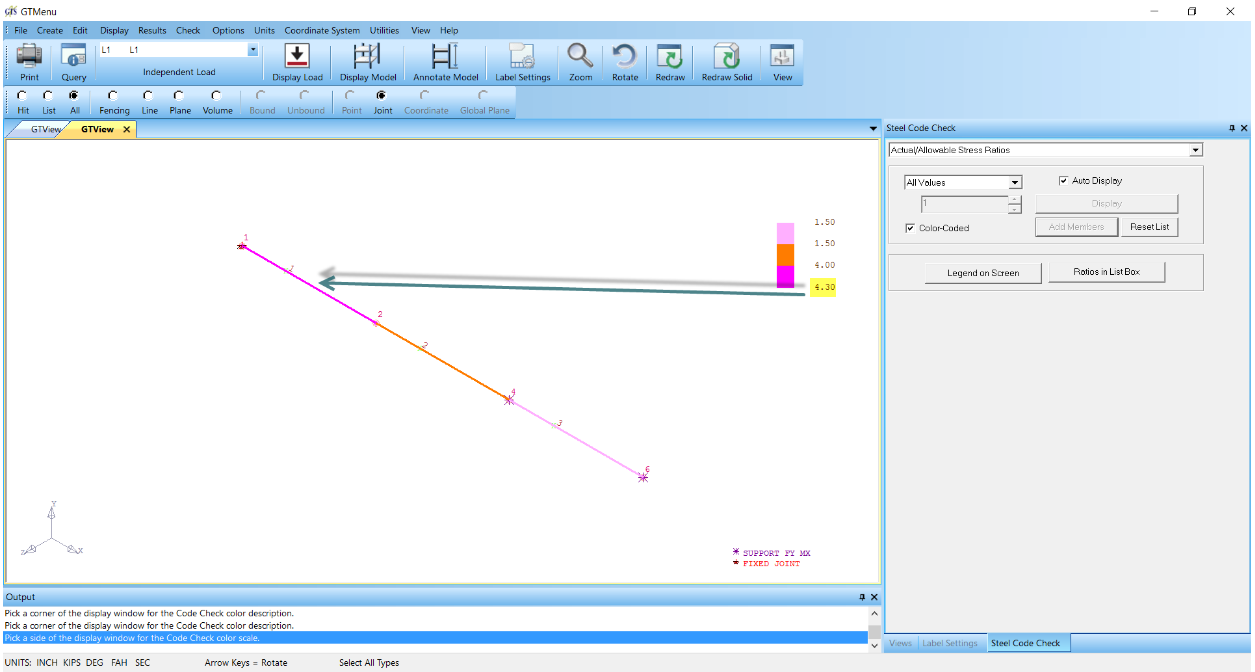
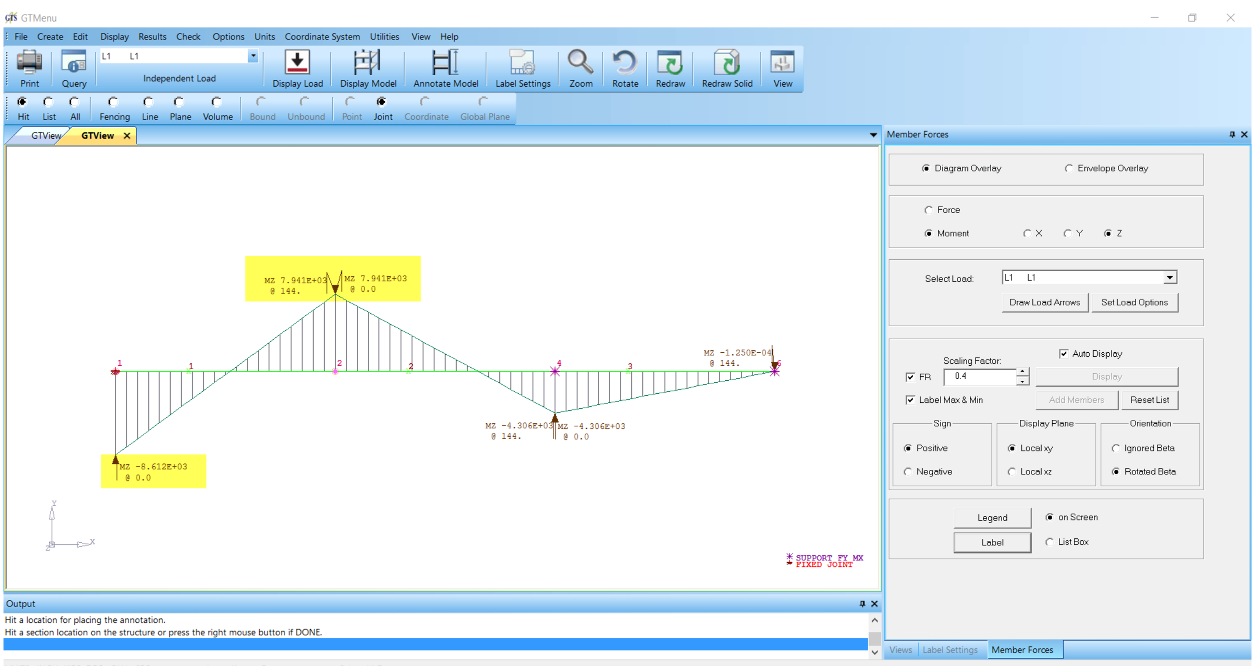
Member	Pass / Fail	Load	Section Location	Critical Ratio	Critical Provision	Current Properties
1	Failed	L1	0.000	4.301	F2-3 LTB	W12x50
2	Failed	L1	0.000	3.966	F2-3 LTB	W12x50
3	Failed	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

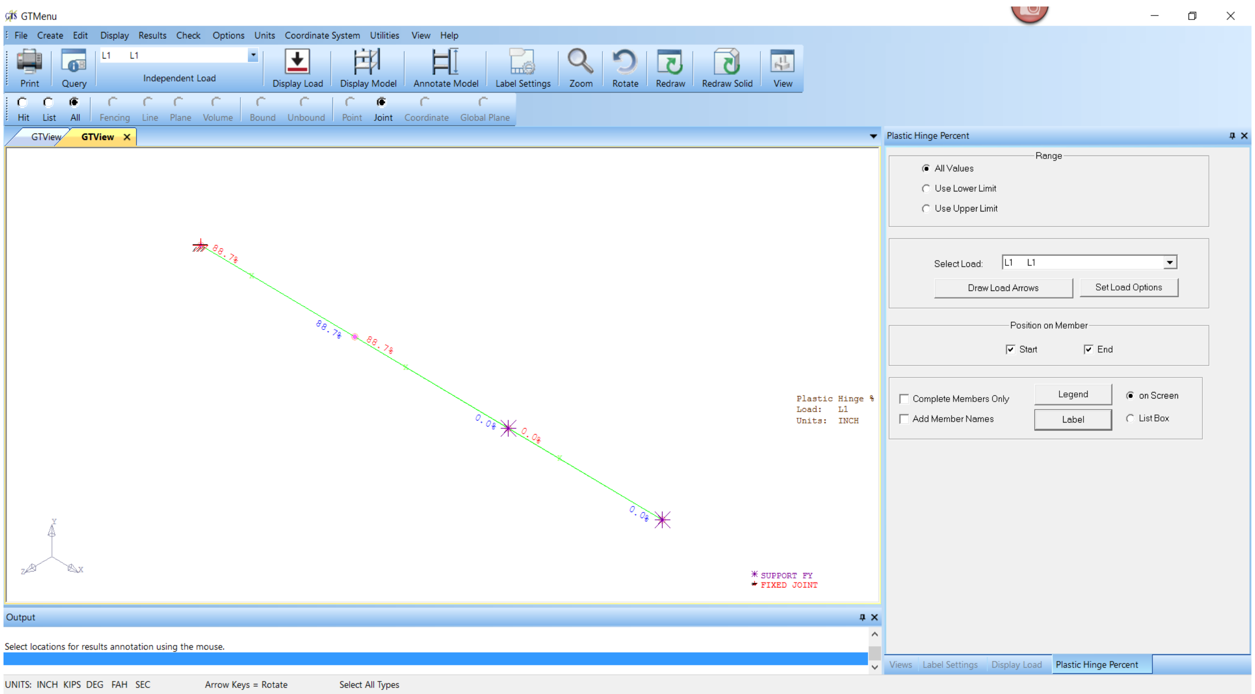


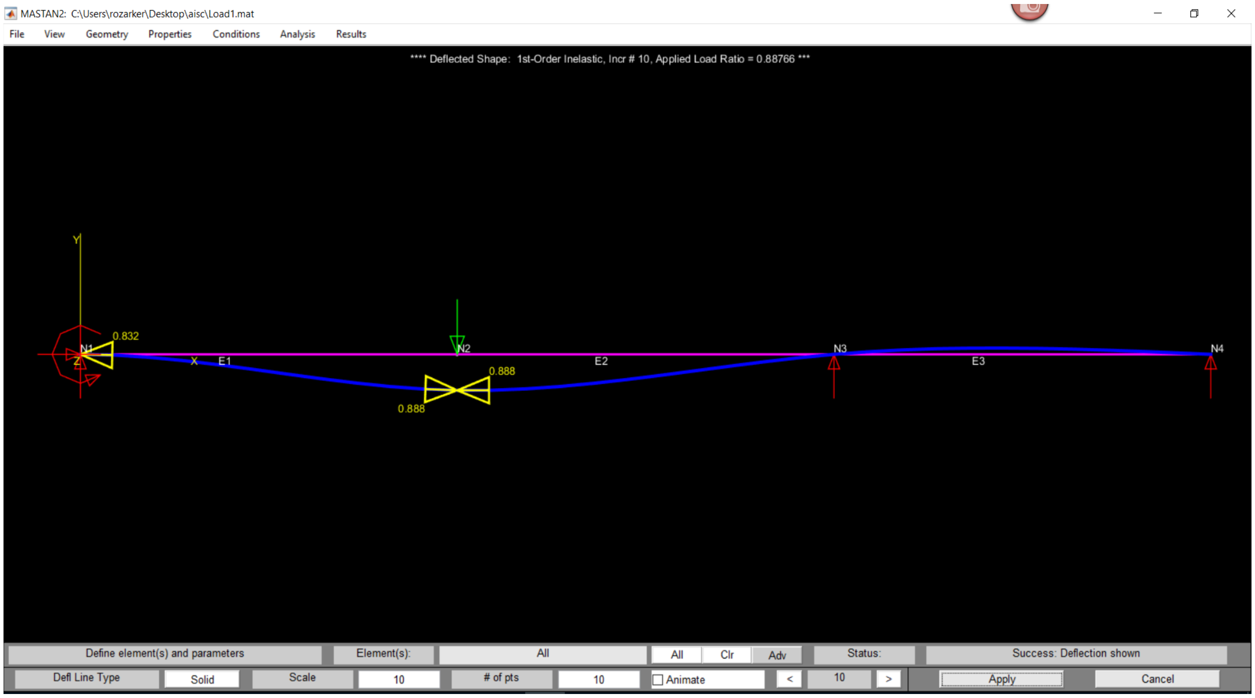


However, we do not have the ability to do a code check in a real structures lab. In a structures lab, we need to establish failure criteria. If this beam were loaded with 200 kips of loading in a structures lab, we would see a catastrophic failure of the beam. This failure most likely will have the formation of plastic zones.

Let us take the same beam in our virtual lab and load it incrementally to that same 100-kip load and monitor the displacements. In this case, the failure criteria would be the formation of a plastic hinge. We would also like to see the behavior of the structure after the plastic hinge is formed.

Both GT STRUDL and Mastan show the formation of plastic hinges at three locations. The hinges at the left and the middle are almost at 89% which means that only 11% more moment can be taken by these joints before a catastrophic failure can occur.

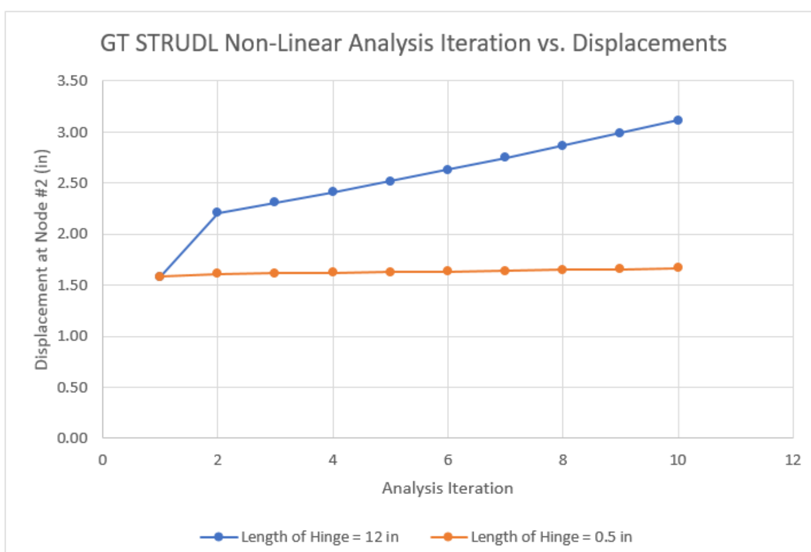




GT STRUDL also offers the option for users to define the length of the plastic hinge, which will affect the displacement results. A beam with a larger plastic zone will deflect more than a beam with a smaller hinge length as shown in the following table.

Table 1: GT STRUDL Non-Linear Analysis Iteration vs. Displacements

Iteration	Displacement at Node #2	
	Inches	Inches
	Length of Hinge = 12 in	Length of Hinge = 0.5 in
1	1.58	1.58
2	2.21	1.61
3	2.31	1.62
4	2.41	1.62
5	2.52	1.63
6	2.63	1.63
7	2.75	1.64
8	2.87	1.65
9	2.99	1.66
10	3.11	1.67



3.1. P-Δ AND P-Δ STABILITY CHECK

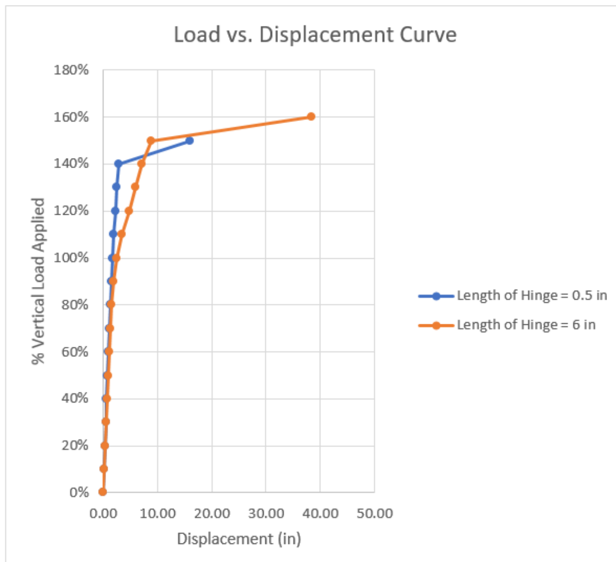
A structure that deforms laterally cannot resist the same gravity loads that an undeformed structure could. A deformed member (i.e. due to deflections caused by applied loads or inherent deformities) will not be able to take the same amount of axial load as a perfectly straight member. These imperfections can cause a catastrophic instability in the entire frame.

The name "P-Δ and P-δ Stability Check" is a name given to the structural analysis procedure which is a part of a more general analysis procedure called "non-linear" structural analysis. Let us examine this using another experiment.

Let us take the same beam member and try loading it until it fails catastrophically. This time we should observe large displacements. Following are the results of this analysis and it is easy to see that no matter what hinge length is used, the beam will fail when loads above 140 kips are applied. This failure is characterized by formation of a hinge mechanism.

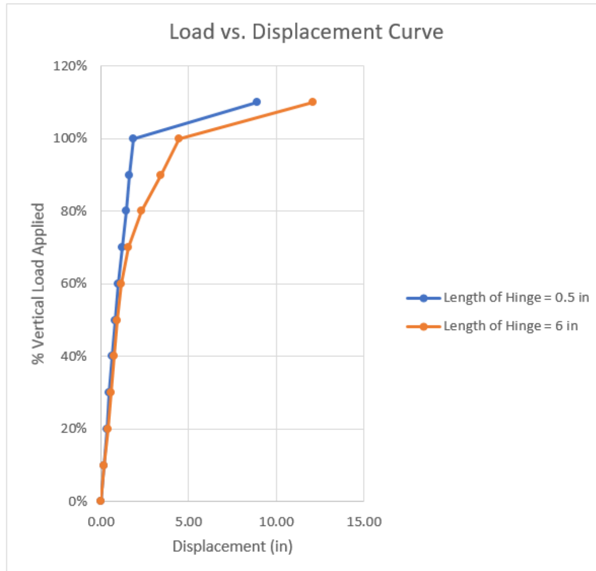
Table 2: GT STRUDL Non-Linear Analysis (Load vs. Displacement Curve)

Iteration	Displacement at Node #2	
	Inches	Inches
	Length of Hinge = 0.5 in	Length of Hinge = 6 in
0%	0.00	0.00
10%	0.16	0.19
20%	0.32	0.38
30%	0.48	0.57
40%	0.64	0.76
50%	0.80	0.95
60%	0.96	1.14
70%	1.13	1.33
80%	1.29	1.52
90%	1.46	1.84
100%	1.67	2.41
110%	1.93	3.38
120%	2.21	4.78
130%	2.46	5.92
140%	2.87	7.20
150%	16.04	8.95
160%		38.42



Let us now examine this same beam with an axial load applied in addition to the load applied at node #2. The load vs. displacement curve for this case is shown below.

Table 2: GT STRUDL Non-Linear Analysis (Load vs. Displacement Curve)		
Iteration	Displacement at Node #2	
	Inches	Inches
	Length of Hinge = 0.5 in	Length of Hinge = 6 in
0%	0.00	0.00
10%	0.16	0.19
20%	0.32	0.38
30%	0.48	0.57
40%	0.64	0.76
50%	0.80	0.95
60%	0.97	1.16
70%	1.21	1.55
80%	1.42	2.30
90%	1.63	3.41
100%	1.84	4.48
110%	8.93	12.12



The load vs. displacement curve shows that the catastrophic failure now occurs at 100 kips compared to 140 kips in the previous loading condition.

This is exactly the type of behavior the codes are worried about which can only be caught by an iterative analysis. In this case, the structure is a simple beam but a common notion in the engineering community is that this type of analysis is only necessary for slender and tall structures is not always true.

3.2. GEOMETRIC IMPERFECTIONS

The code also requires engineers to consider the effects of accidental manufacturing defects that can result in a slight global imperfection. For example, before analysis is performed, an out-of-plumb geometry of h/500 is purposely induced using a load applied in the horizontal direction. This load is equal to 0.002 * self-weight of the structure. This is called notional load and needs to be considered in all load combinations without creating favorable effects.

Our experiment above suggested that the presence of an axial load resulted in an earlier formation of plastic hinges and subsequent failure. The opposite is also true. A column subjected to pure axial load will be able to sustain a lot more load than a column that is subjected to a lateral load or has an inherent manufacturing defect.

3.3. STIFFNESS REDUCTION

The code requires engineers to consider the effects of presence of residual stresses when steel is hot rolled. Hot steel sections are cooled after they have gone through significant casting processes. The cooling process although gradual is never uniform throughout the length and depth of the section which causes internal stresses. These stresses remain in the section which are termed as residual stresses. A section already subjected to residual stresses will not be able to take ideal stresses calculated by engineers per the code. Hence, stiffness reduction factors are assigned to all members by reducing EI or EA during analysis.

The axial load in the experiment discussed above induces stresses in the member. Hence the capacity of the beam to take bending stresses is reduced. The same would be true if residual stresses are present in the members when they are installed. As it is hard to predict what these stresses are the code suggests that the stiffness reduction of 0.8EI and 0.8 EA be applied to all members.

4. CONCLUSION

The lingering question that engineers ask after these codes are implemented is "***Are my designs performed as per AISC ASD 9th Ed. Or older (i.e. Green Book) incorrect***?"

No, they are not. If you observe the combined axial and bending clause in AISC ASD 9th Ed., the Fa (Axial Capacity) was calculated after considering K values calculated per the nomographs, which was a tedious process.

When $f_a/F_a \leq 0.15$, Equation (H1-3) is permitted in lieu of Equations (H1-1) and (H1-2):

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad \text{(H1-3)}$$

The newer code relives the engineer from calculating the K values which increases the axial capacity (i.e. P_c) in the following equation but also increases the M_{rx} the M_{ry} (calculated applied moment) terms in the numerators because of second order effects (both P-Δ and P-δ), stiffness reduction, and notional loads.

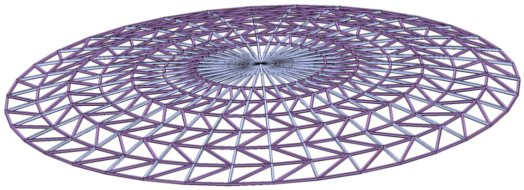
(a) When $\frac{P_r}{P_c} \geq 0.2$

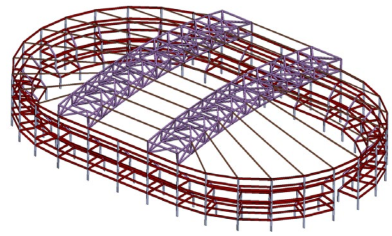
$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

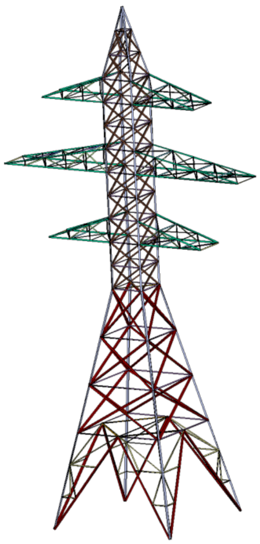
(b) When $\frac{P_r}{P_c} < 0.2$

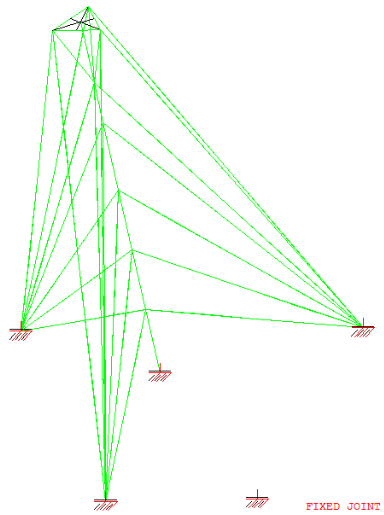
$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

The advantage of the newer code is that an engineer is now directly modeling effects that can affect structural systems, members, and member cross-sectional instability leveraging advanced analysis software products like GT STRUDL. Not only does this remove the limitations required by K factor analysis, but also simplifies the analysis and design of any structure with a complex geometry such as a spherical steel tank roof, stadium roofs, and architectural monuments. All of these are easily analyzed, designed and code checked using advanced general-purpose structural analysis tools like GT STRUDL.









5. REFERENCE

1. Course Introduction and Modern Methods of Structural Analysis, from Linear to Nonlinear – Part II, American Institute of Steel Construction (AISC) Night School, Ronald D. Ziemian, P.E., Ph.D., 2015
2. Matrix Structural Analysis, John Wiley and Sons, Inc., William McGuire, Second Ed., 2000
3. Performance-Based Plastic Design, SEA and ICC, Subhash Goel, 2008
4. Steel Construction Manual 14th Ed., American Institute of Steel Construction (AISC), 2011
5. Steel Construction Manual 9th Ed., American Institute of Steel Construction (AISC), 1989

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